Count-Min and Augmented Sketch

Sketches for data streams

[Cormode & Muthukrishnan, J. Algorithms 55, 2005]
[Roy, Khan & Alonso, ACM SIGMOD 2016]

Recommended:
[Metwally, Agrawal & Abbadi, 2005]
[Roy, Teubner & Alonso, ACM SIGKDD 2012]

(slides by Patrick Dinklage, released under CC0)

Sketches

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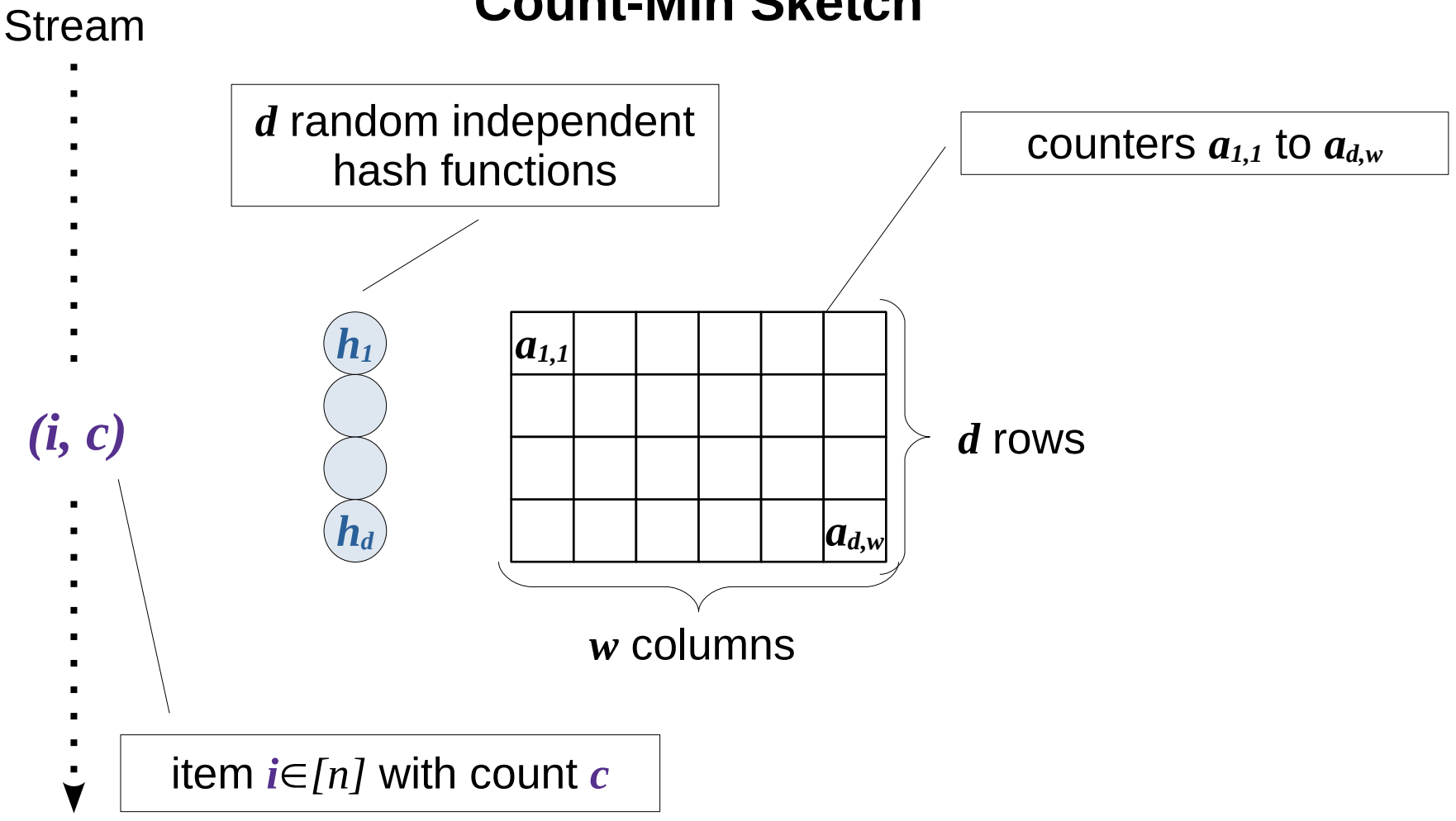
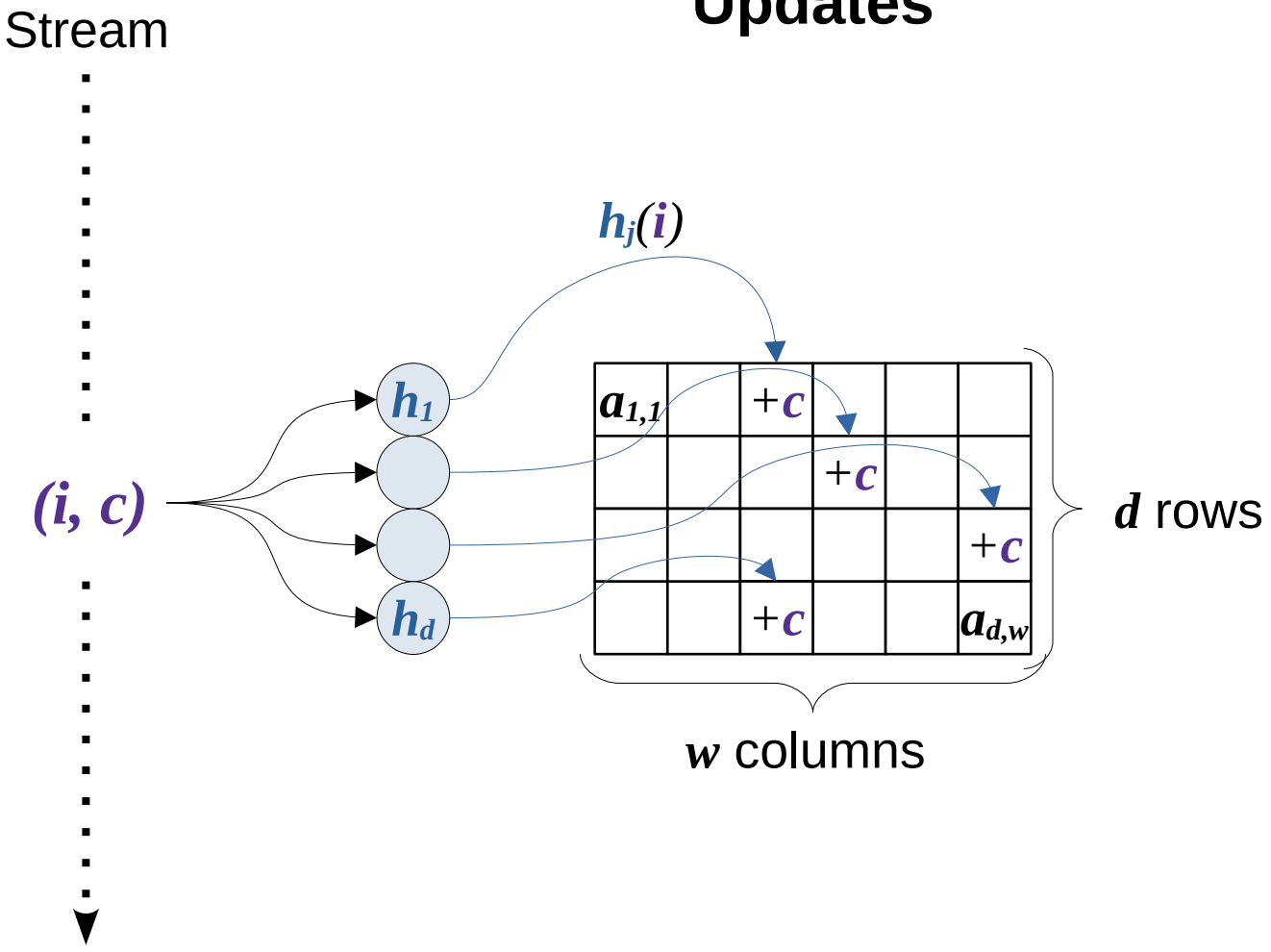
  - Much less space (polylog) needed than for explicit representation
- Commonly of interest: *frequency counters*
  - Frequency estimation, quantiles, heavy hitters, …
- Reasonable error margin w.h.p. for queries

Count-Min Sketch



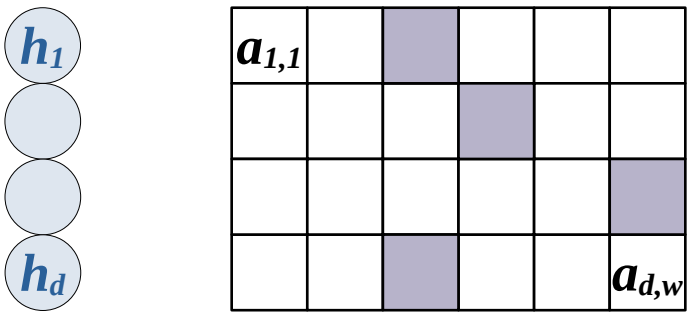
Updates



Point Queries

(non-negative case)

$$Q(i) := \min\{a_{j,h\_j(i)} \mid 1 \le j \le d\}$$



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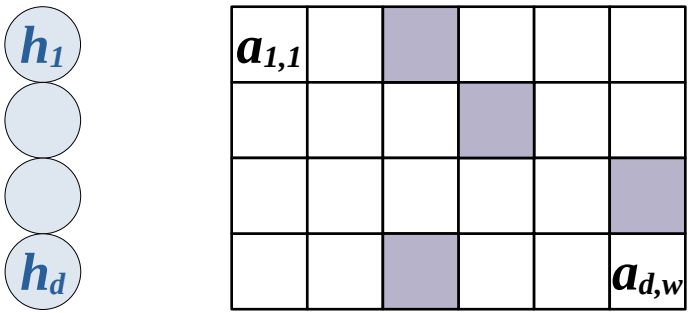

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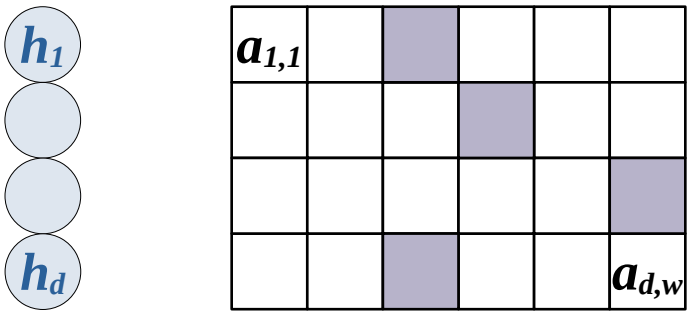

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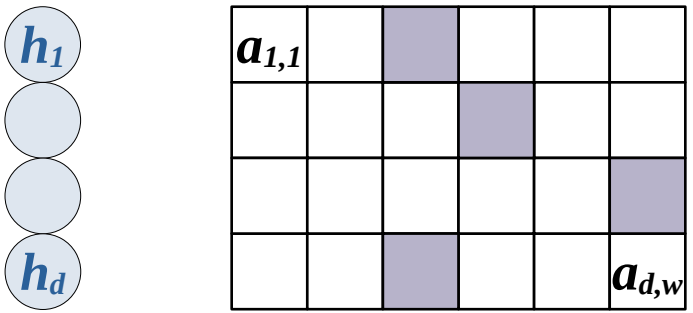

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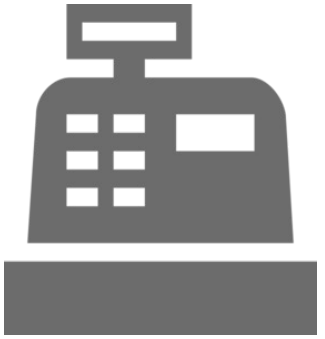
  2) $Q(i) \le f_i + \varepsilon ||f||_1$ with probability at least $1-\delta$

Heavy Hitters in the Cash Register



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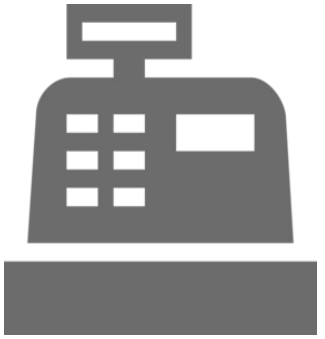
Heavy Hitters in the Cash Register



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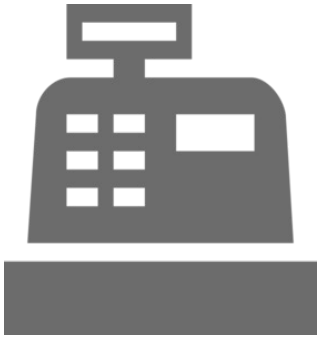

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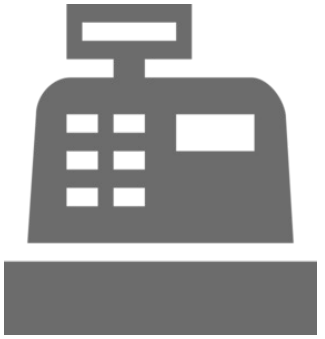

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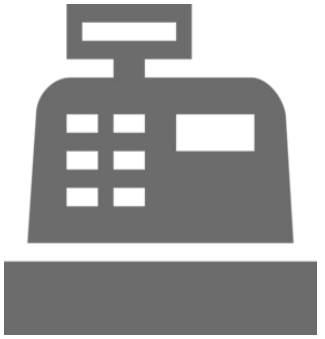

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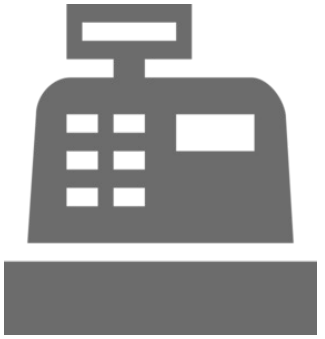

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1) H contains all items i with frequency $f_i \geq \phi||f||_1$

2) With probability $1-\delta$, H contains no items i' with $f_{i'} \leq (\phi-\varepsilon)||f||_1$

Motivation for Augmented Sketch

- Errors due to frequency over-estimation

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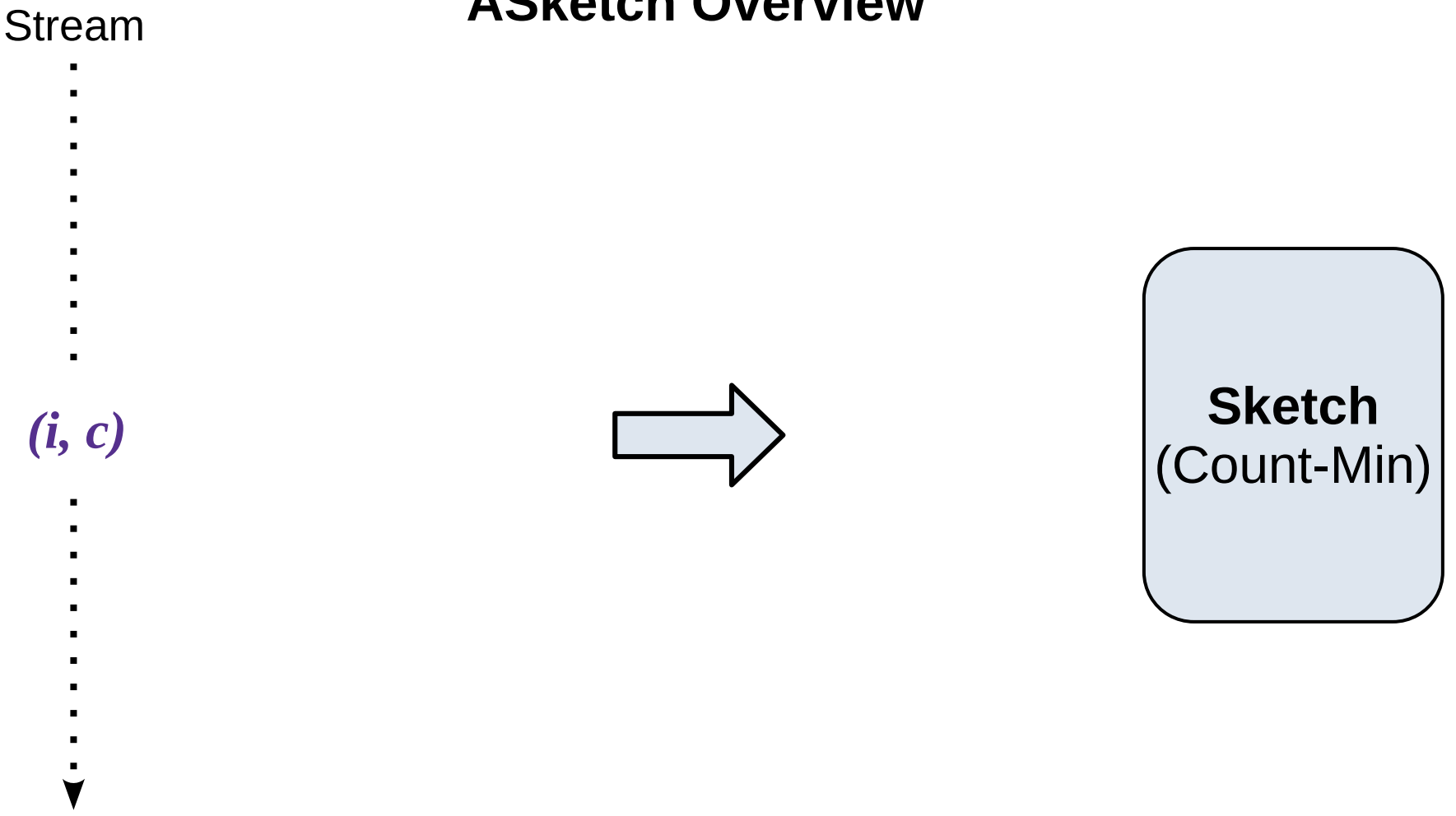
  - Count-Min: 0.0024% → **ASketch**: 0.0004%
- False positives
  - Count-Min: few → **ASketch**: (practically) none
- Performance
  - **ASketch** throughput is 4-5 times higher than Count-Min

ASketch Overview



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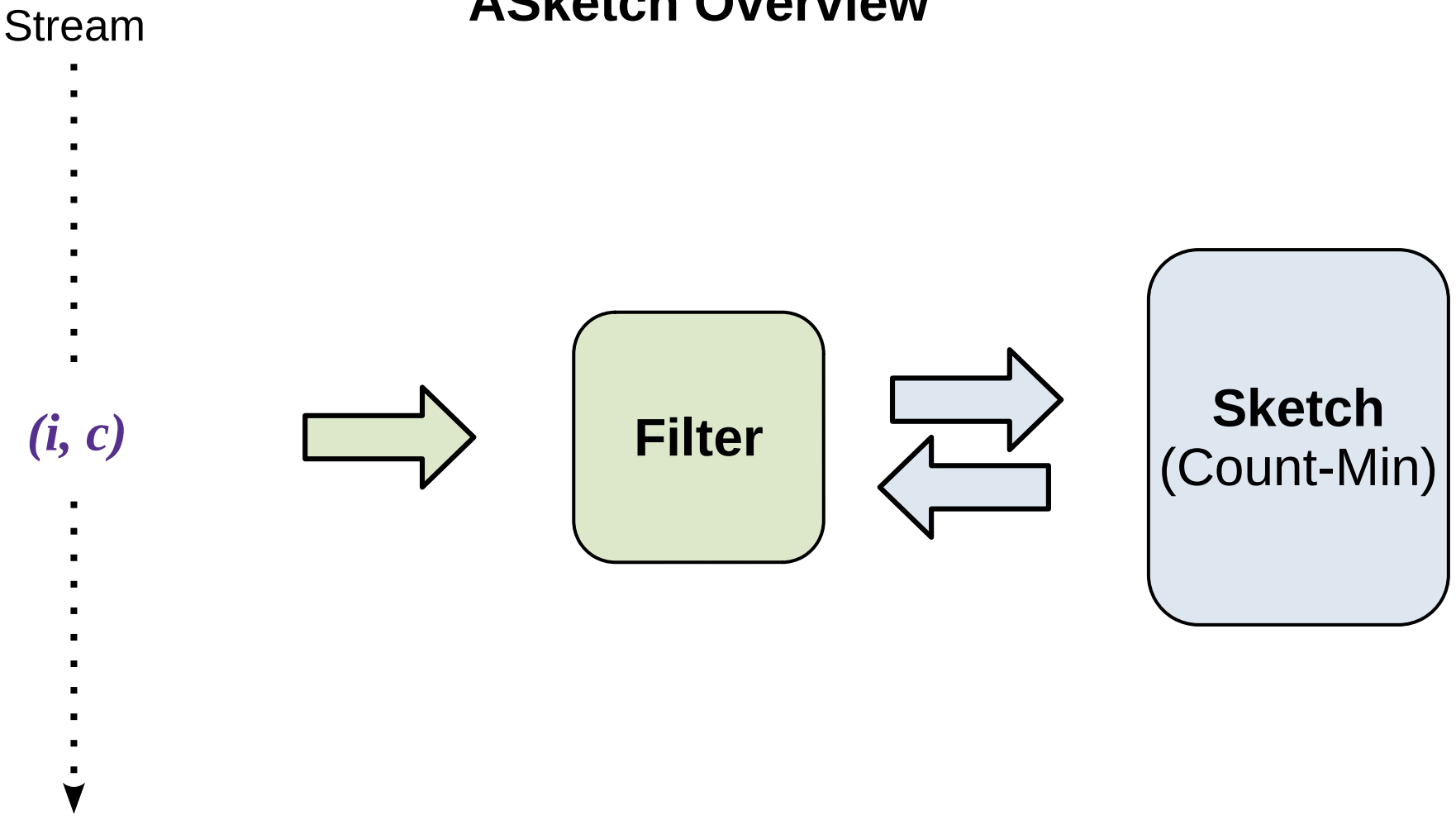

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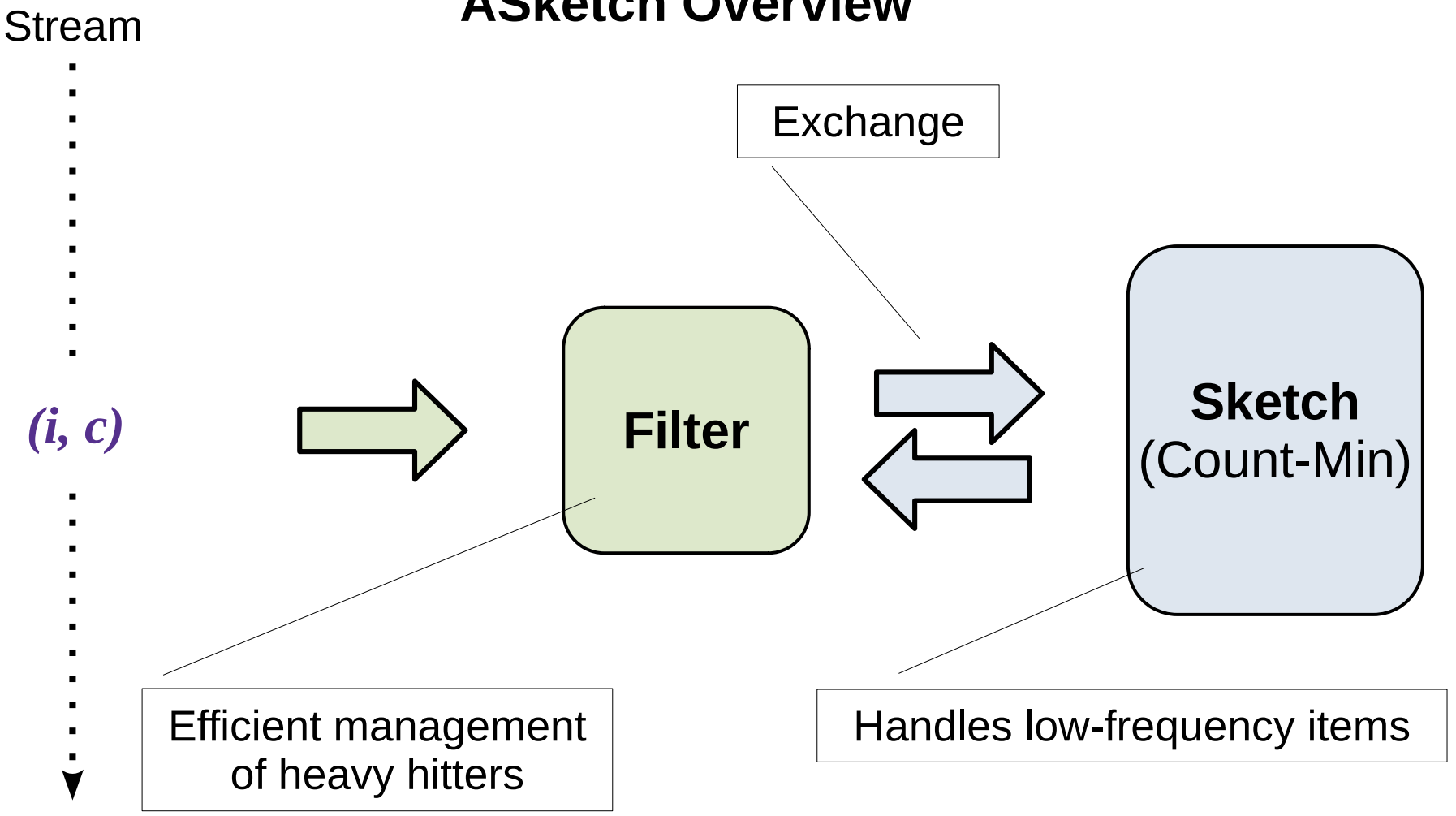

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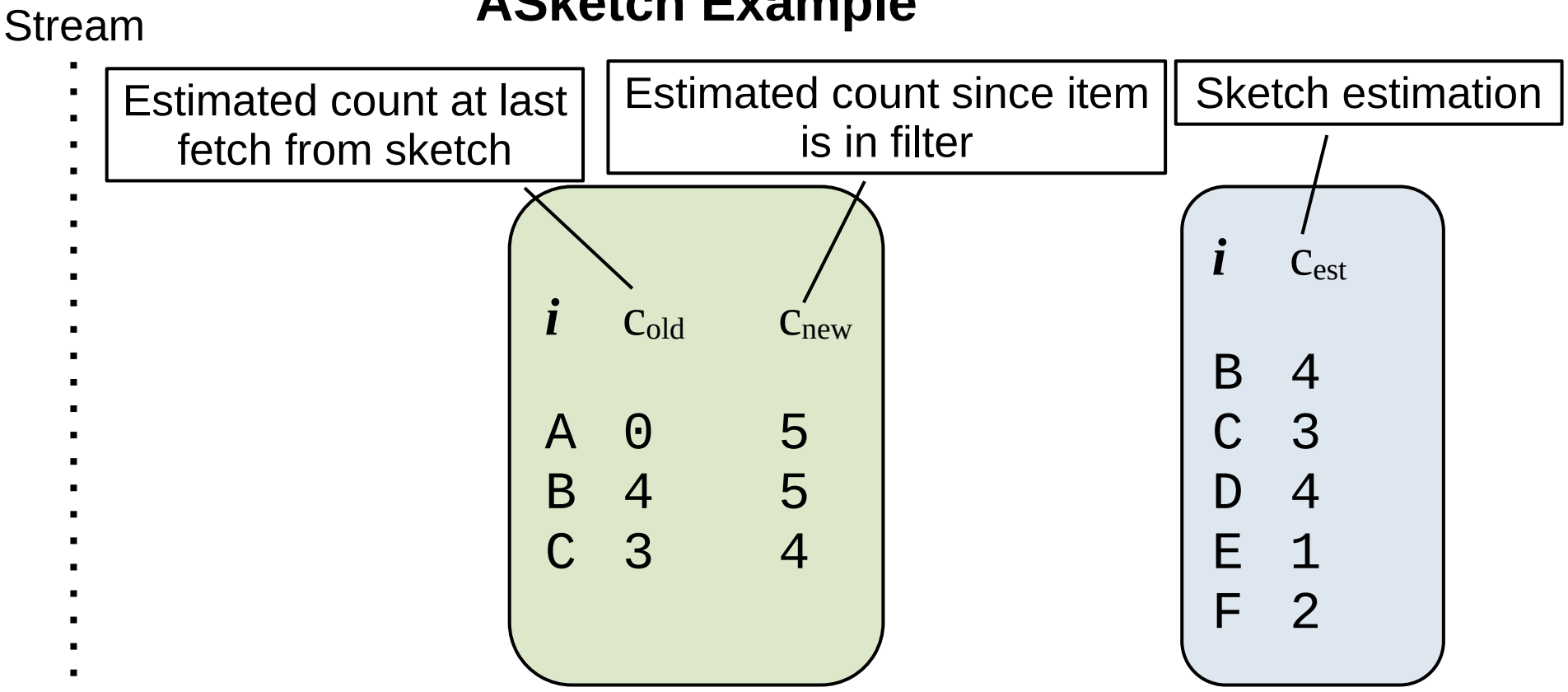
- Additional small error for low-frequency items is tolerable

ASketch Example



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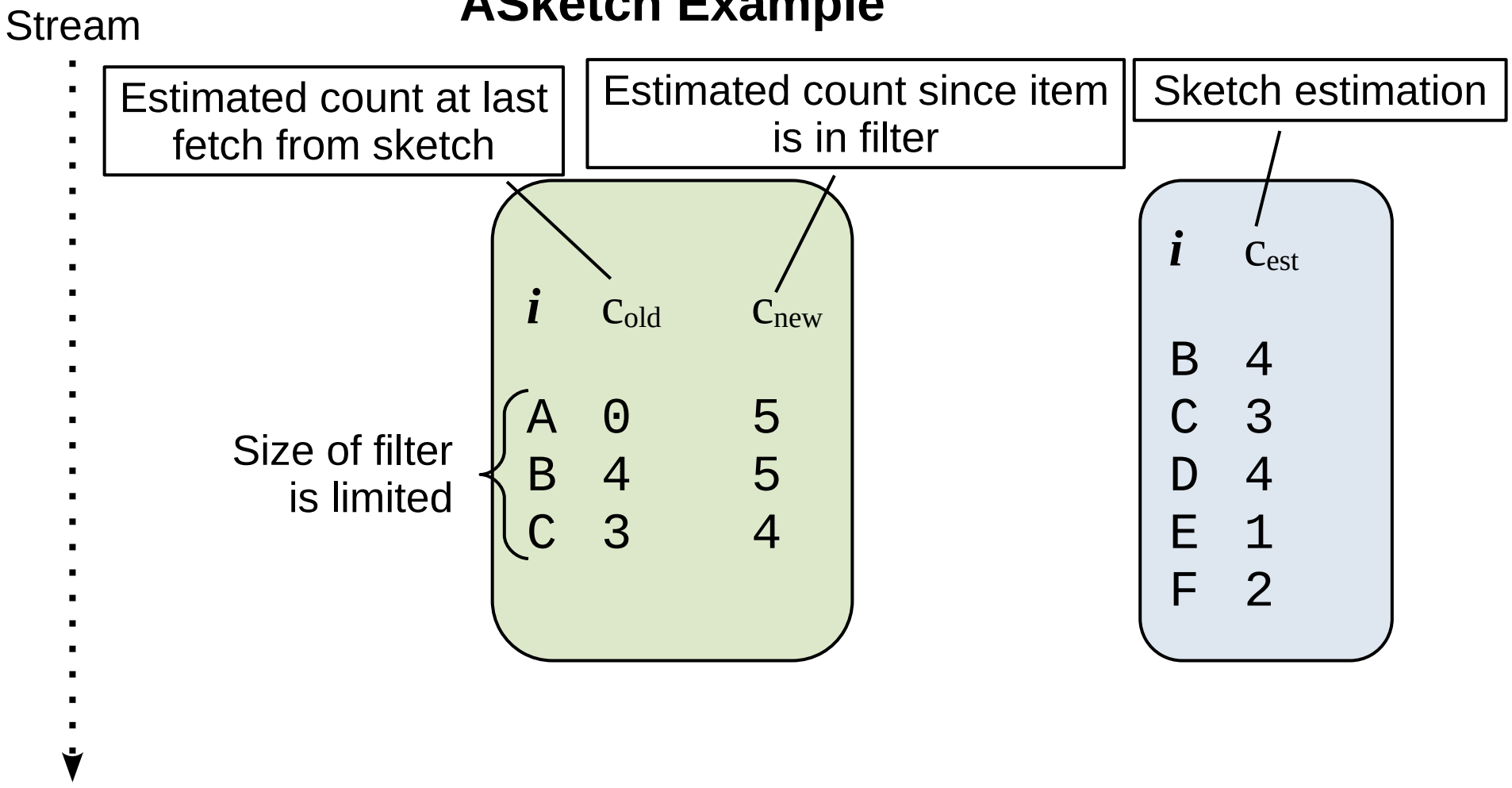

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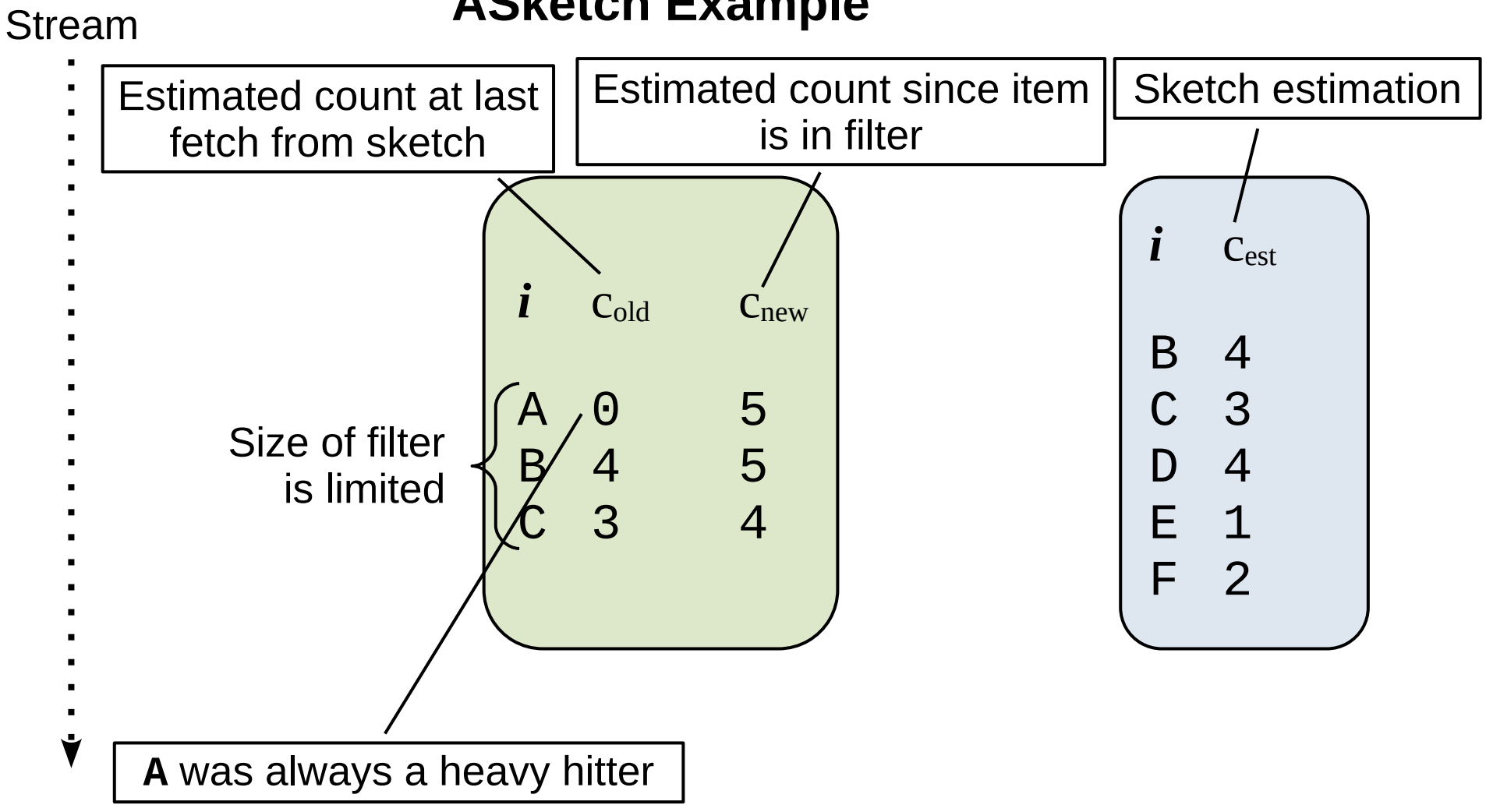

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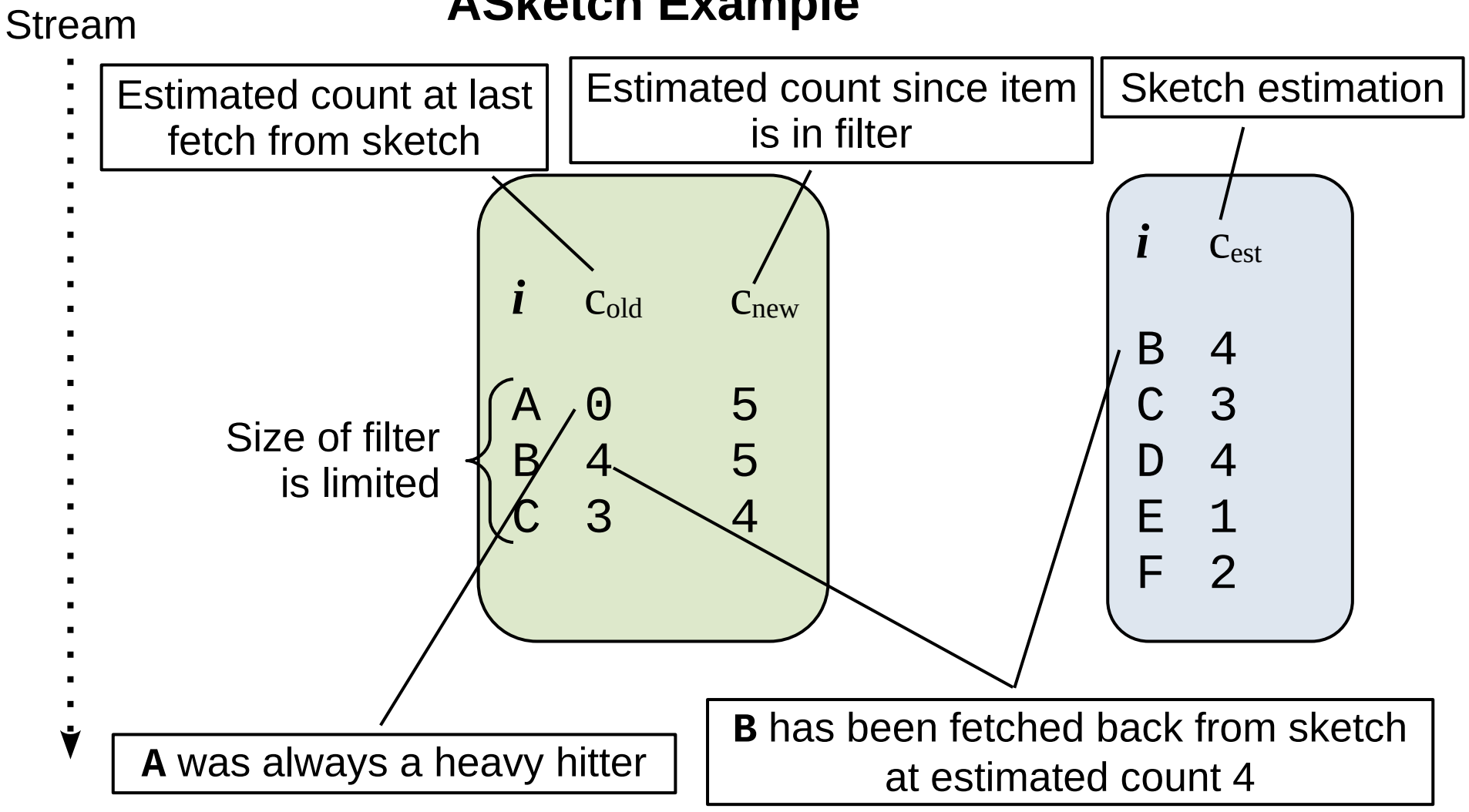

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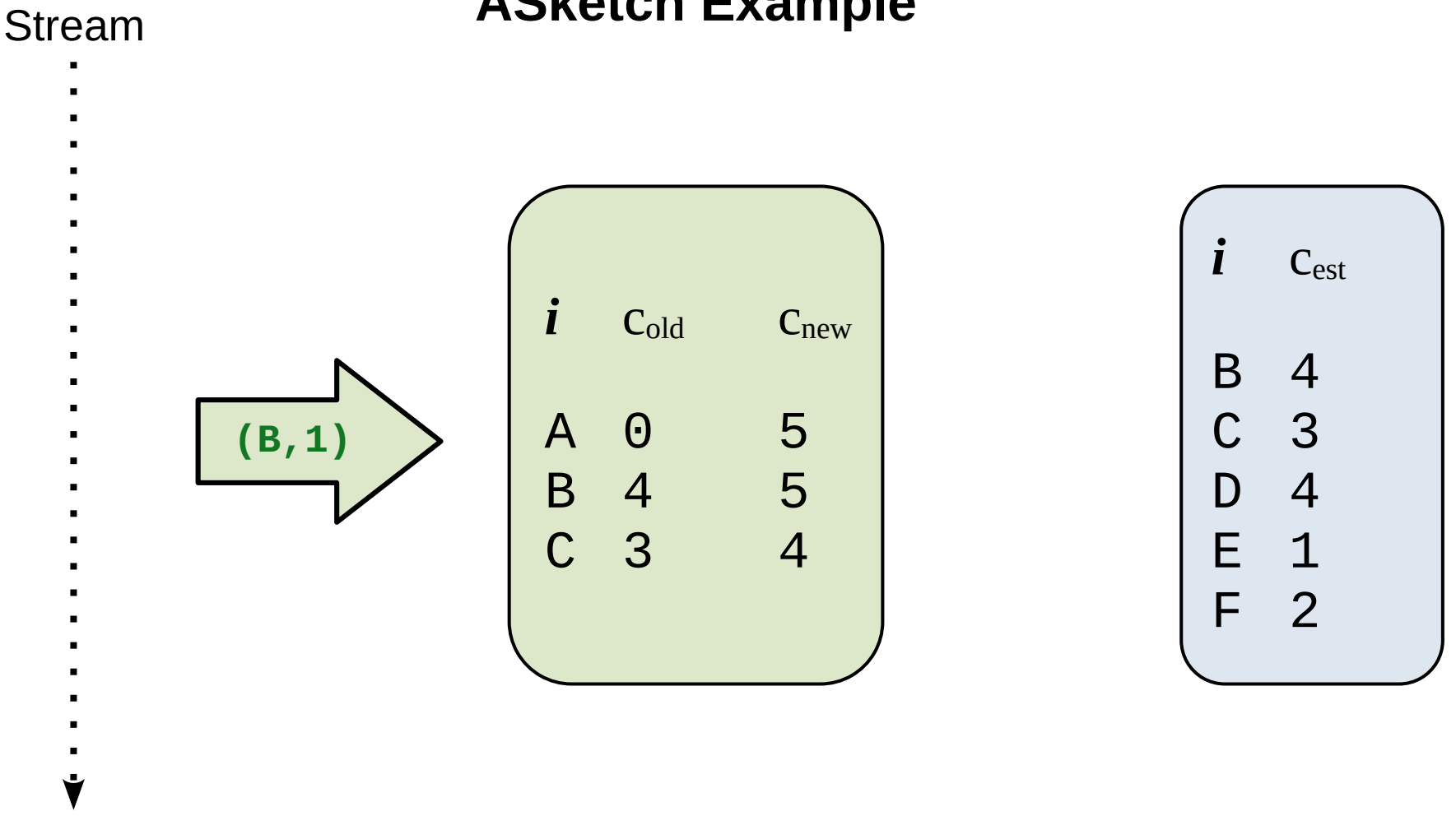

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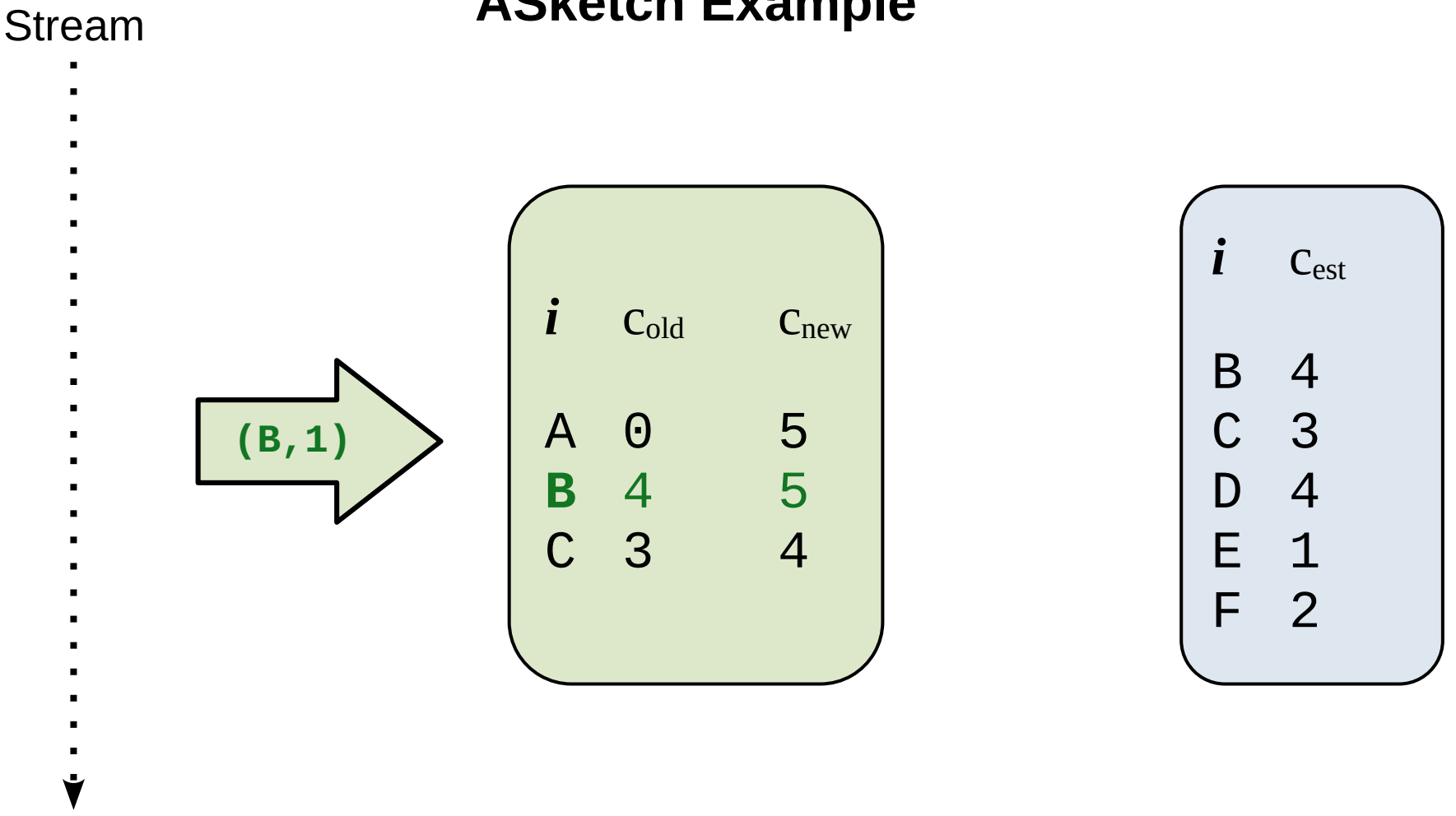

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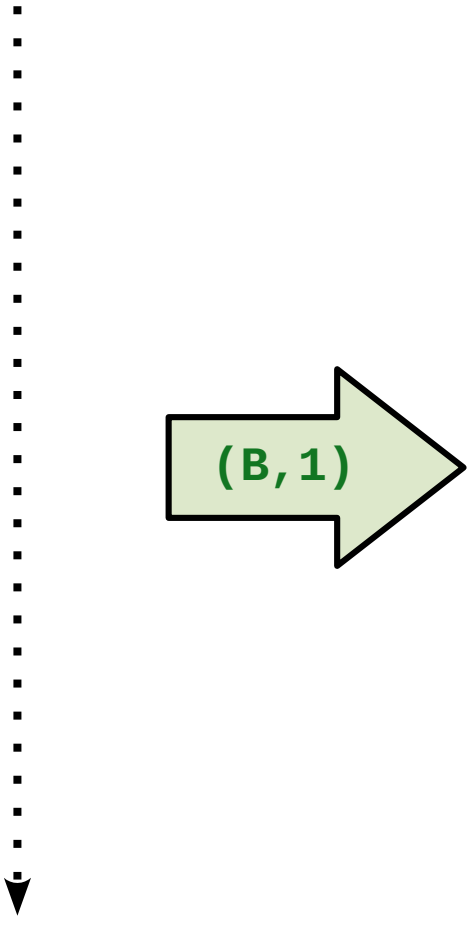

(B,1)

i	C_{old}	C_{new}
A	0	5
B	4	**6**
C	3	4

i	C_{est}
B	4
C	3
D	4
E	1
F	2

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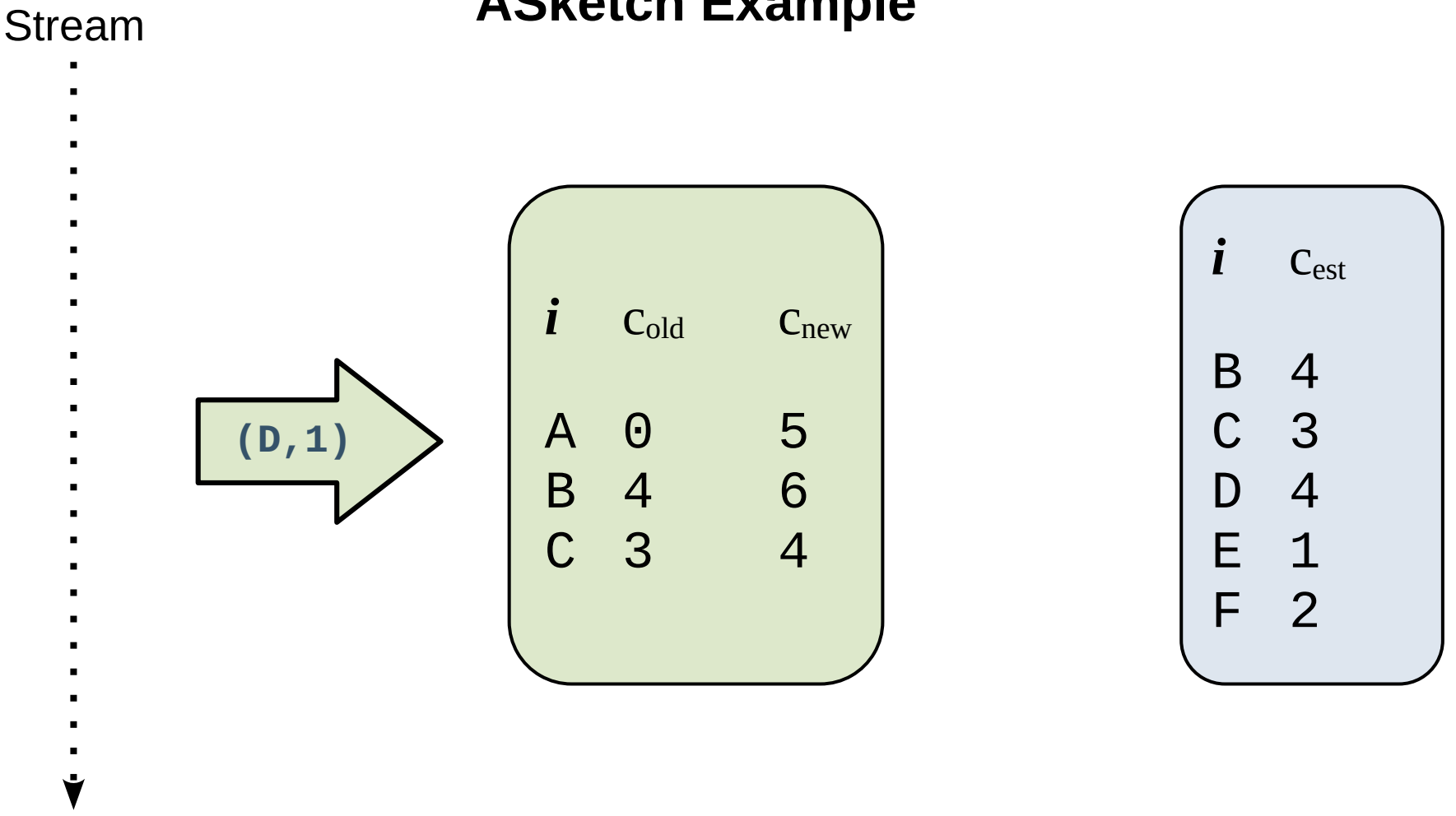

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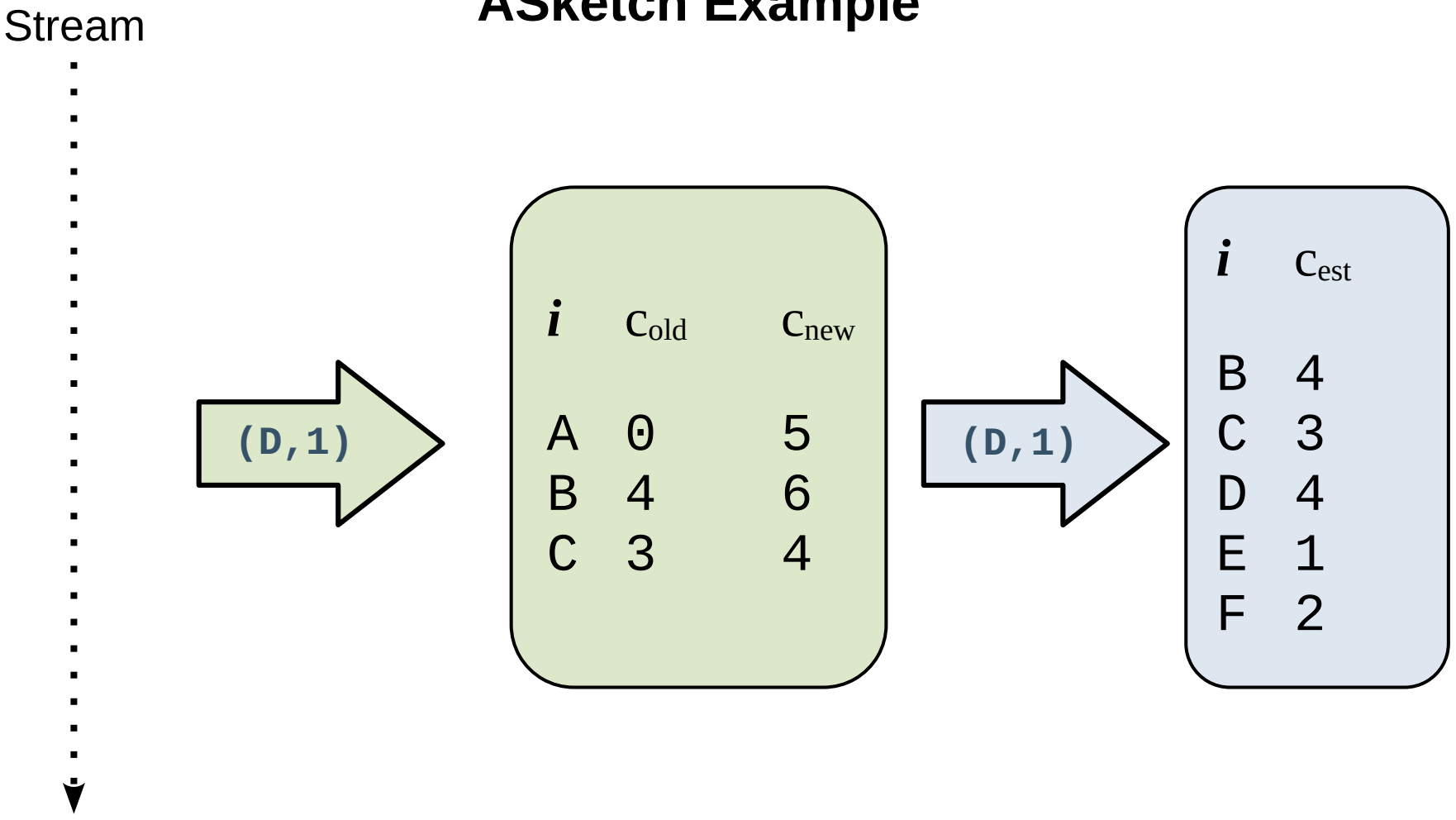

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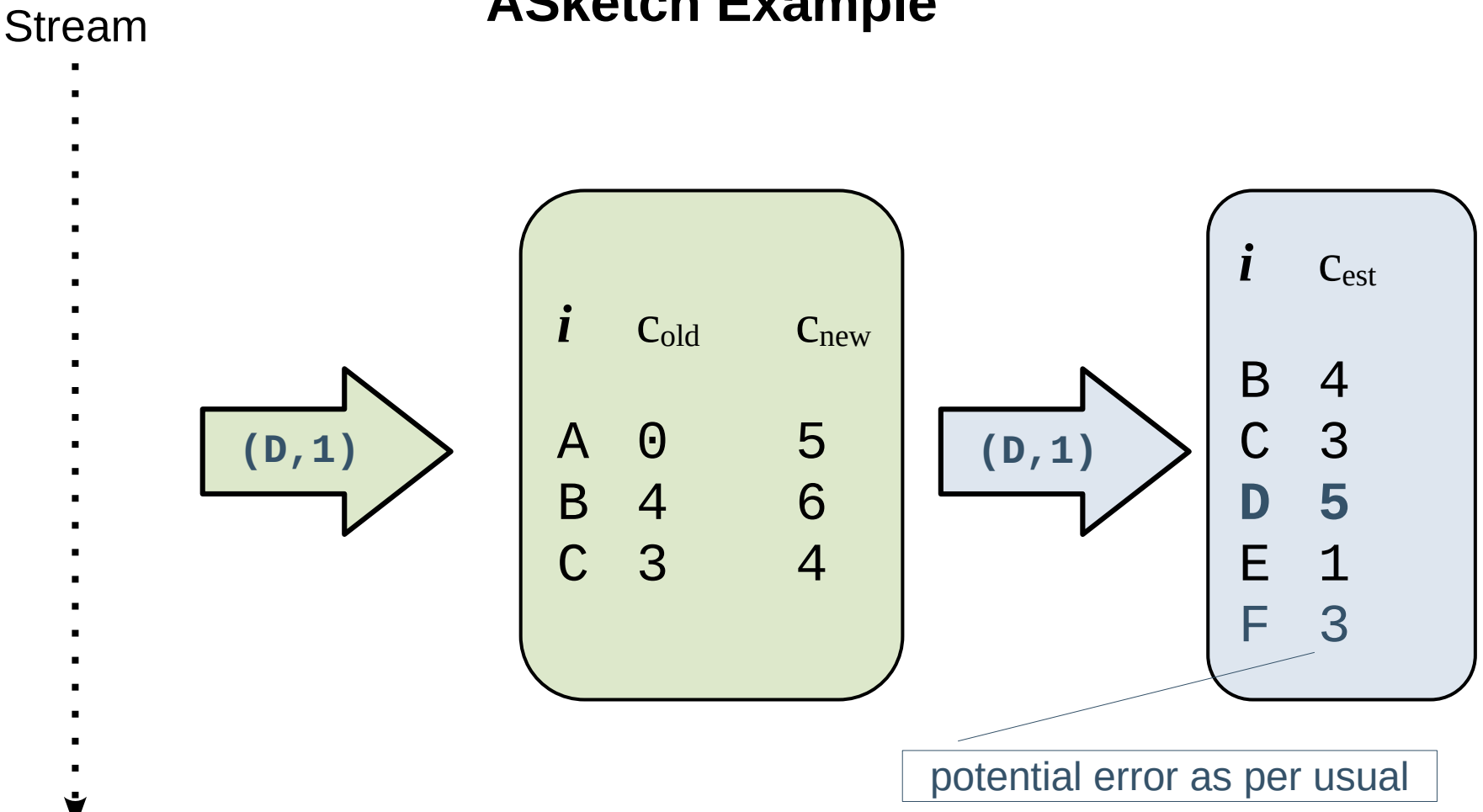

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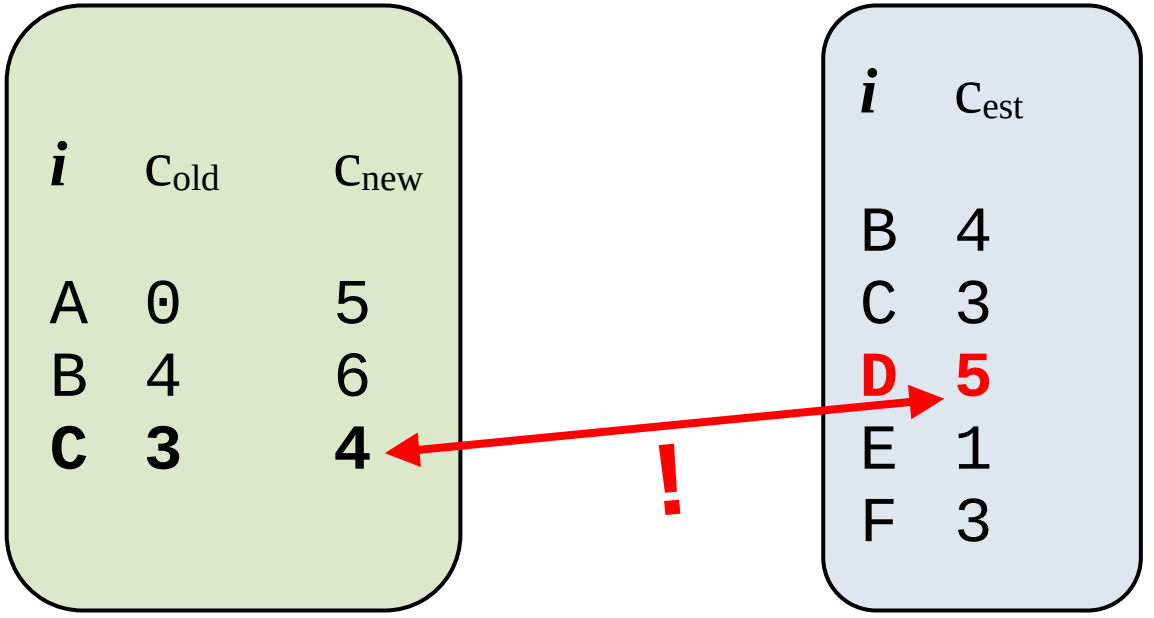

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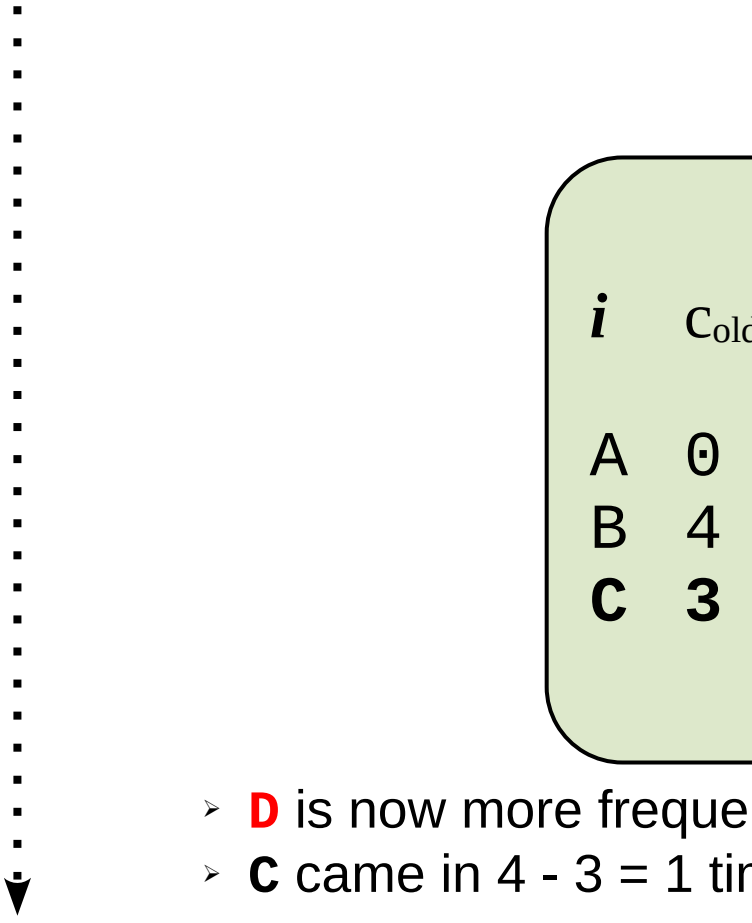

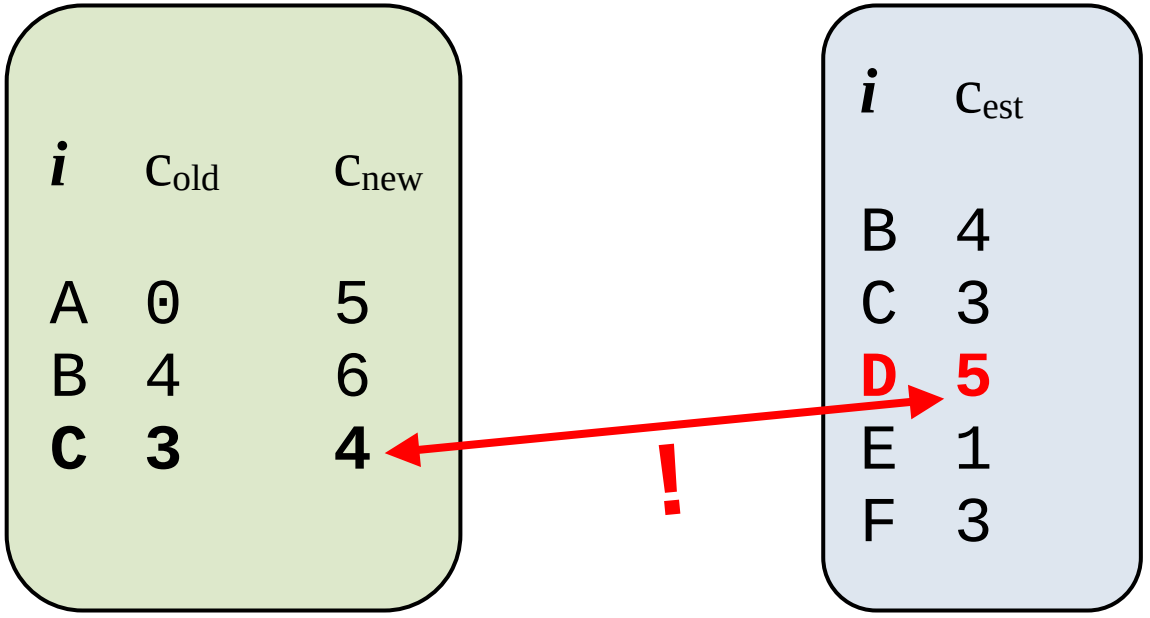

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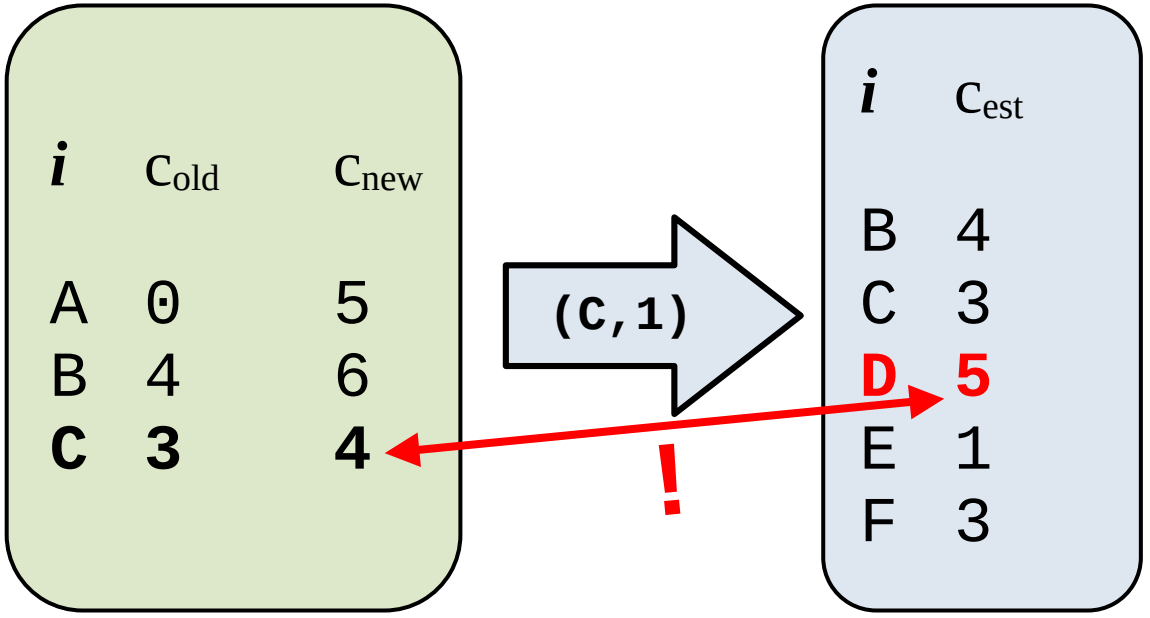

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C	**4**
D	**5**
E	**2**
F	3

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- **This may introduce additional error in the sketch!**

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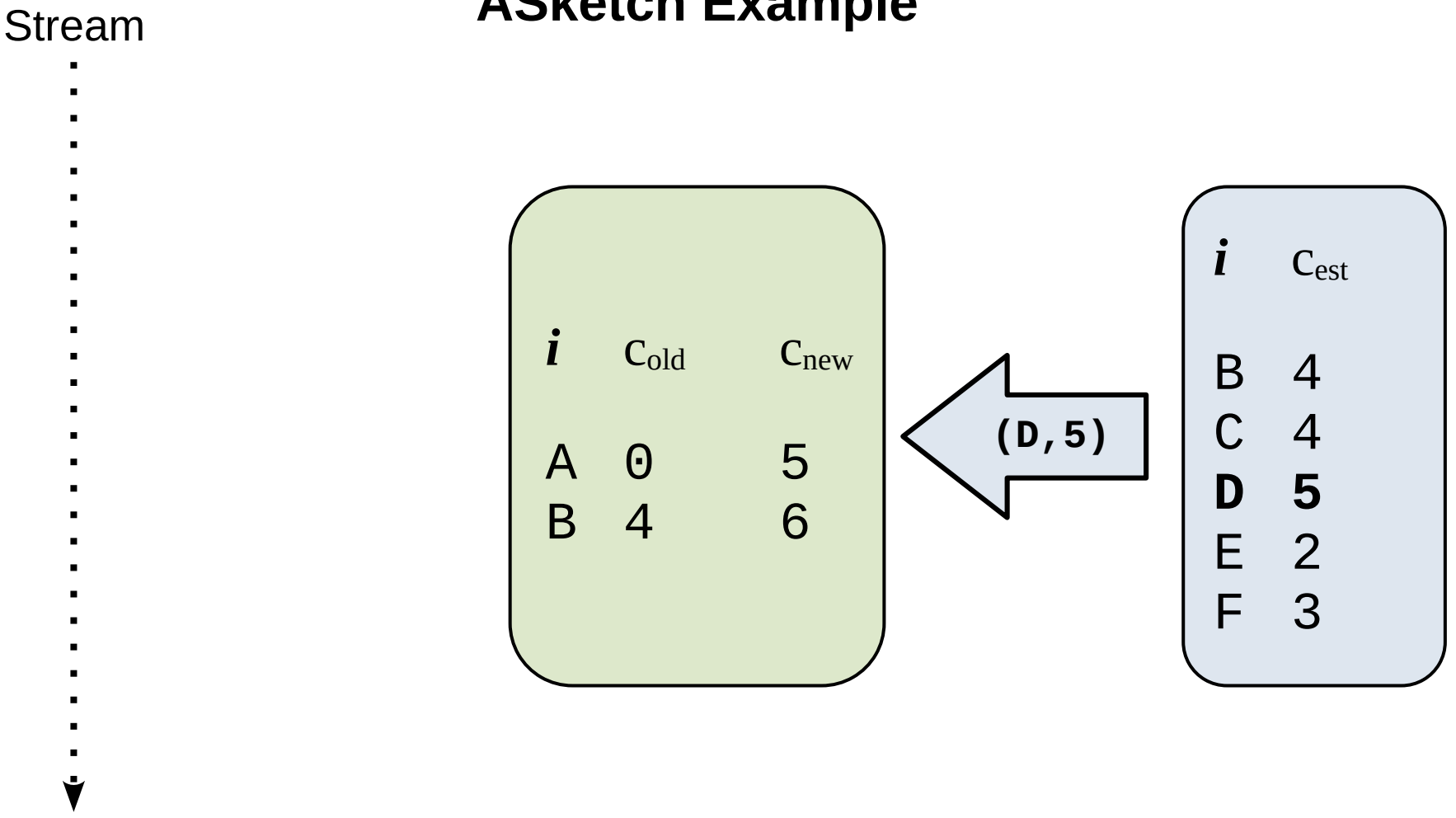

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- ASketch is suitable for *pipeline parallelism*